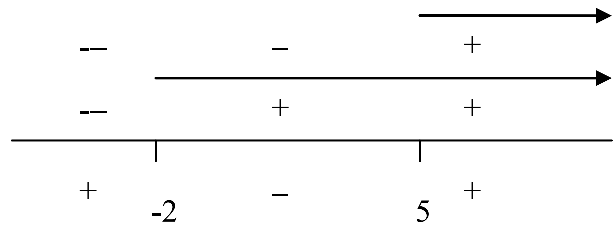
Marking Scheme
PEPERIKSAAN PERCUBAAN STPM NEGERI PAHANG 2012
Mathematics T Paper 1 (954/1)/ Mathematics S Paper 1 (950/1)
Set 1

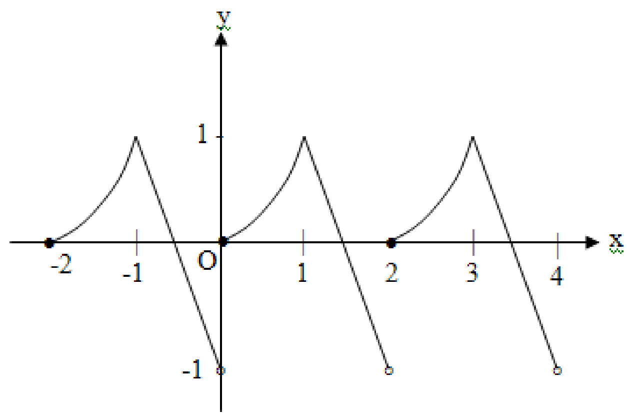
<table>
<tr><th>NO.</th><th>SCHEME</th><th>MARKS</th></tr>
<tr><td>1.</td><td>$[(A' \cap C) \cup (B' \cap C)] \cup (A \cap B \cap C)$
$= [(A' \cup B') \cap C) \cup (A \cap B \cap C)$
$= [(A' \cup B') \cup (A \cap B)] \cap C$
$= [(A \cap B)' \cup (A \cap B)] \cap C$
$= \xi \cap C$
$= C$</td><td>B1
B1
B1

B1</td></tr>
<tr><td></td><td></td><td>[4 marks]</td></tr>
<tr><td>2.</td><td>$h = \frac{2-1}{4} = 0.25$
<table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>0.6931</td></tr><tr><td>1.25</td><td>0.9410</td></tr><tr><td>1.5</td><td>1.1787</td></tr><tr><td>1.75</td><td>1.4018</td></tr><tr><td>2</td><td>1.6094</td></tr></table>$\int_1^2 \ln(1+x^2)\,dx = \frac{0.25}{2}[\,0.6931+1.6094 + 2(0.9410+1.1787+1.4018)]$
$= 1.168$</td><td>B1

B1

M1
A1</td></tr>
<tr><td></td><td></td><td>[4 marks]</td></tr>
</table>

NO.	SCHEME	MARKS
3.	(a) $\frac{1}{x-5} - \frac{1}{x+2} \geq 0$ $\frac{7}{(x-5)(x+2)} \geq 0$  The solution set = {x : x < -2 or x > 5, x ∈ R } (b) $-(x+5) < 3x-2 < x+5$ $-x-5 < 3x-2$ and $3x-2 < x+5$ $-3 < 4x$ and $2x < 7$ $-\frac{3}{4} < x$ and $x < \frac{7}{2}$ $\{x : -\frac{3}{4} < x < \frac{7}{2}\}$	B1 M1 A1 M1 M1 A1
		[4 marks]
4.	$e^x y = \sin x$ $e^x \frac{dy}{dx} + e^x y = \cos x$ $e^x (\frac{dy}{dx} + y) = \cos x$ $e^x (\frac{d^2y}{dx^2} + \frac{dy}{dx}) + (\frac{dy}{dx} + y)e^x = -\sin x$ $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} + y = -y$ $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0.$	M1A1 M1A1 M1 A1
		[6 marks]

NO.	SCHEME	MARKS
5.	(a) $\lim\limits_{x\to1^+} f(x) = \lim\limits_{x\to1^+}(3-2x) = 1$	B1
	$\lim\limits_{x\to1^-} f(x) = \lim\limits_{x\to1^-} x^2 = 1$	B1
	$f(1) = 3-2(1) = 1$	M1
	$\lim\limits_{x\to1} f(x) = f(1)$	A1
	$\therefore f$ is continuous at $x = 1$.	
	(b)	
		D1 (curve) D1 (line segment) D1 (All correct)
		[7 marks]
6.	(a) $z^2 = p^2 - 9 - 6pi$	B1
	(b) $\dfrac{[(p^2-9)-6pi](4+3i)}{(4-3i)(4+3i)}$ [multiplied by conjugate complex]	M1
	$\dfrac{4(p^2-9)+18p}{25} + \dfrac{3(p^2-9)-24p}{25}i$	A1
	$\dfrac{3(p^2-9)-24p}{25} = 0$	M1
	$p^2 - 8p - 9 = 0$	
	$p = 9$ or $p = -1$	A1
	(c) $\dfrac{z^2}{4-3i} = -2$	A1
	(d) $\arg(-1-3i) = \tan^{-1}\dfrac{-3}{-1}$	M1
	$= -(\pi - \tan^{-1}3)$	A1
	$= -1.893$ rad	
		[8 marks]

NO.	SCHEME	MARKS
7.	(a)(i) $3x^2+3y^2-6x+12y-60=0$	
	$x^2-2x+y^2+4y=20$	M1
	$(x-1)^2-(-1)^2+(y+2)^2-(2)^2=20$	
	$(x-1)^2+(y+2)^2=5^2$	
	centre = $(1\,,-2)$ and radius = 5 units	A1A1
	(a)(ii) distance from centre of C to straight line l	
	$=\dfrac{\lvert (1)-2(-2)+5\rvert}{\sqrt{1^2+(-2)^2}}$	M1
	$=2\sqrt{5}$ units	A1
	(b)(i) $3x^2+3y^2-6x+12y-60=0$, $2y=x+5$	
	$3(2y-5)^2+3y^2-6(2y-5)+12y-60=0$	M1
	$y^2-4y+3=0$	
	$(y-1)(y-3)=0$	M1
	$y=1$ or $y=3$	
	$x=-3$ or $x=1$	
	$A(-3\,,1)$, $B(1\,,3)$	A1
	(b)(ii) $k=\dfrac{5}{2}$	B1
		[9 marks]

NO.	SCHEME	MARKS
8.	(a) $f(x) = 3x$	B1
	(b) $\dfrac{1-x^2}{x^3+x} \equiv \dfrac{1-x^2}{x(x^2+1)}$	
	Let $\dfrac{1-x^2}{x(x^2+1)} \equiv \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$	
	then $1-x^2 \equiv A(x^2+1)+(Bx+C)(x)$	B1
	When $x = 0$, $A = 1$	
	By equating coefficients of $x : C = 0$	M1A1
	By equating coefficients of $x^2 : A + B = -1 \Rightarrow B = -2$	
	$\therefore \dfrac{1-x^2}{x(x^2+1)} \equiv \dfrac{1}{x} - \dfrac{2x}{x^2+1}$	A1
	(c) $\int_1^2 \dfrac{3x^4+2x^2+1}{x^3+x}\,dx = \int_1^2 \left(3x + \dfrac{1}{x} - \dfrac{2x}{x^2+1}\right) dx$	B1
	$= \left[\dfrac{3x^2}{2} + \ln\lvert x\rvert - \ln\lvert x^2+1\rvert\right]_1^2$	M1
	$= (6+\ln 2-\ln 5)-(\dfrac{3}{2}+0-\ln 2)$	M1
	$= \dfrac{9}{2} + \ln\left(\dfrac{4}{5}\right)$	A1
		[9 marks]

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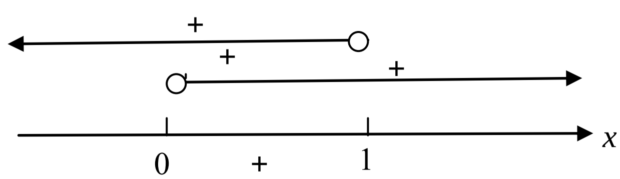
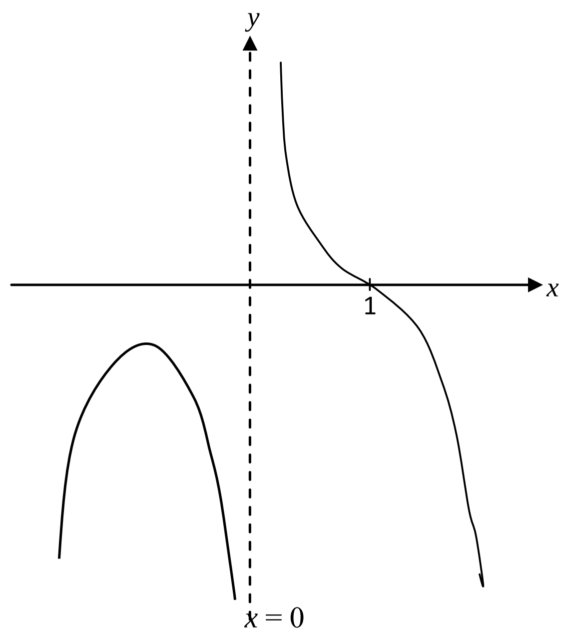
NO.	SCHEME	MARKS
9.	. $(1-x)^{-2} = 1+\frac{-2}{1!}(-x)+\frac{(-2)(-3)}{2!}(-x)^2+\frac{(-2)(-3)(-4)}{3!}(-x)^3+\cdots$	M1
	$= 1+2x+3x^2+4x^3+\cdots$	
	$(1+ax^2)^{\frac{1}{4}} = 1+\frac{\frac{1}{4}}{1!}(ax^2)+\frac{(\frac{1}{4})(-\frac{3}{4})}{2!}(ax^2)^2$	
	$= 1+\frac{1}{4}ax^2+\cdots$	A1 (either one series)
	$\frac{\sqrt[4]{1+ax^2}}{(1-x)^2} = (1+ax^2)^{\frac{1}{4}}(1-x)^{-2}$	
	$= (1+\frac{1}{4}ax^2+\cdots)(1+2x+3x^2+4x^3+\cdots)$	M1 (multiply both correct series)
	$= 1+2x+(3+\frac{1}{4}a)x^2+(4+\frac{1}{2}a)x^3+\cdots$	A1
	(a) $3+\frac{1}{4}a=4$ or $4+\frac{1}{2}a=6$	M1
	$a=4$	A1
	(b) $\lvert -x\rvert<1$ and $\lvert 4x^2\rvert<1$	M1
	$\{x: -\frac{1}{2}<x<\frac{1}{2}\}$	A1
	(c)	
	$\frac{\sqrt[4]{1+4x^2}}{(1-x)^2} \approx 1+2x+4x^2+6x^3$	
	$\frac{\sqrt[4]{1+4(\frac{1}{8})^2}}{(1-\frac{1}{8})^2} \approx 1+2(\frac{1}{8})+4(\frac{1}{8})^2+6(\frac{1}{8})^3$	M1
	$\frac{\sqrt[4]{\frac{17}{16}}}{(\frac{7}{8})^2} \approx 1.32422$	
	$\sqrt[4]{17} \approx 2.02771$	
	$\sqrt[4]{17} \approx 2.03\ (3\ s.f.)$	A1
		[10 marks]

NO.	SCHEME	MARKS
10.	(a) $\frac{dy}{dx} = -\frac{1}{x^2} - 2x = 0$	
	$x = \sqrt[3]{-\frac{1}{2}} = -0.794$	M1
	Since there is only one real value of x, $\therefore$ there is only one turning point	A1
	$\frac{d^2y}{dx^2} = \frac{2}{x^3} - 2 = -6$	M1
	This point is a maximum point.	A1
	(b) $\frac{d^2y}{dx^2} = \frac{2}{x^3} - 2 < 0$	
	$\frac{2-2x^3}{x^3} < 0$	M1
	$2 - 2x^3 > 0$ and $x^3 > 0$	
	$\therefore\ x < 1$ $\quad$ $\therefore x > 0$	
		M1
	Curve concaves downward for $\{x: x < 0\ or\ x > 1\}$	A1
	Curve concaves downward for $\{x : 0 < x < 1\}$	A1
	(c) 	D1 Curve in 3rd quadrant with max. pt. D1 Curve with point of inflexion (1,0) and concavity in correct direction D1 Curve approaches y-axis as asymptote
		[11 marks]

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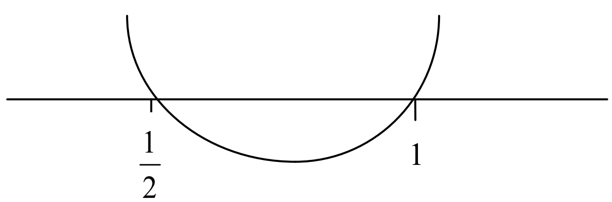
NO.	SCHEME	MARKS
11.	(a) $p\left(\frac{1}{2}\right)=6\left(\frac{1}{2}\right)^4+a\left(\frac{1}{2}\right)^3+b\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)-1=0$	M1
	$a+2b=1$	A1
	$p'(x)=24x^3+3ax^2+2bx+1$	
	$p'(-1)=24(-1)^3+3a(-1)^2+2b(-1)+1=-76$	M1
	$3a-2b=-53$	A1
	$a=-13,\ \ b=7$	M1A1
	(b) $p(x)=6x^4-13x^3+7x^2+x-1$	
	1 is zero of $p(x)=6x^4-13x^3+7x^2+x-1$	
	because $p(1)=6(1)^4-13(1)^3+7(1)^2+1-1=0$	B1
	$p(x)=(2x-1)(x-1)Q(x)$	M1
	$(2x-1)(x-1)(x-1)(3x+1)=0$	A1
	$x=\frac{1}{2},\ 1,-\frac{1}{3}$	A1
	(c) $\frac{(2x-1)(x-1)(x-1)(3x+1)}{(x-1)(3x+1)}\leq 0$	
	$(2x-1)(x-1)\leq 0$ and $x\neq-\frac{1}{3}$, $x\neq 1$	M1
		M1
	The solution set = $\{x:\frac{1}{2}\leq x<1\}$	A1
		[13 marks]

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| NO. | SCHEME | MARKS |
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| 12.(a) | (a) $\lvert M \rvert = 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix}$ | M1 |
| | $= 4$ | |
| | Since $\lvert M \rvert \neq 0$, M is not singular | A1 |
| | (b) N – 6M = $\begin{pmatrix} 3 & -1 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 4 \end{pmatrix}$  M(N – 6M) = $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ | B1 B1 |
| | M(N – 6M) = 4I<br>N – 6M = $4M^{-1}I$<br>$M^{-1}$ = 1/4 (N – 6M) | M1 |
| | $= \frac{1}{4}\begin{pmatrix} 3 & -1 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 4 \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{4} & \frac{1}{2} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix}$ | A1 |
| | (c) $adjoinM = \lvert M \rvert \times M^{-1}$ | M1 |
| | = N – 6M | |
| | $= \begin{pmatrix} 3 & -1 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 4 \end{pmatrix}$ | A1 |
| | (d) $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 \\ 30 \\ 20 \end{pmatrix}$ or $\begin{pmatrix} 20 & 0 & -10 \\ 0 & 20 & 10 \\ -10 & 10 & 20 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -100 \\ 300 \\ 200 \end{pmatrix}$ | B1 |
| | $M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -10 \\ 30 \\ 20 \end{pmatrix}$ | |
| | $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 3 & -1 & 2 \\ -1 & 3 & -2 \\ 2 & -2 & 4 \end{pmatrix}\begin{pmatrix} -10 \\ 30 \\ 20 \end{pmatrix}$ | M1 |
| | $= \begin{pmatrix} -5 \\ 15 \\ 0 \end{pmatrix}$ | A1 |
| | x = -5. y = 15, z = 0 | A1 |
| | | **[13 marks]** |
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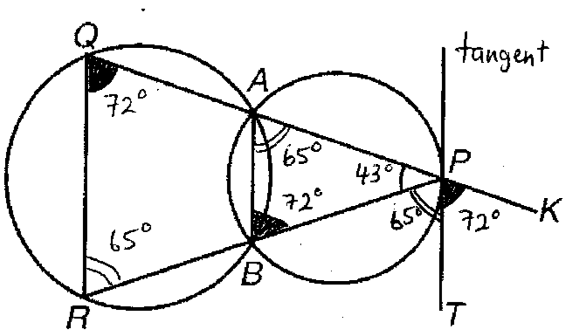
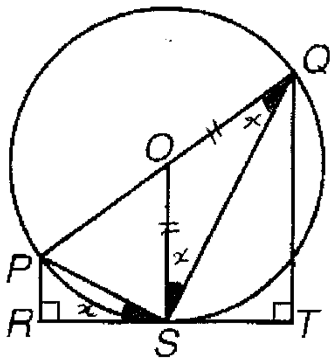
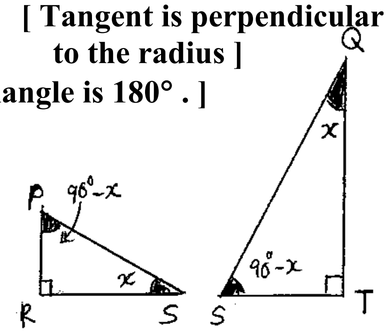
Marking Scheme
PEPERIKSAAN PERCUBAAN STPM NEGERI PAHANG 2012
Mathematics T Paper 2 (954 / 2) Set 1

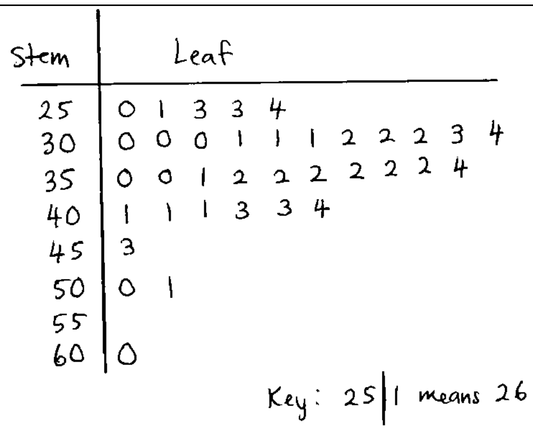
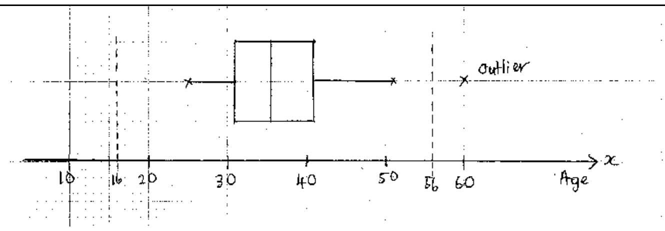
Q	Mark Scheme	Marks
1	$\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta$	
	$= [\sin 5\theta + \sin\theta] + [\sin 7\theta + \sin 3\theta]$	B1 Factor formula
	$= 2\sin 3\theta\cos 2\theta + 2\sin 5\theta\cos 2\theta$	
	$= 2\cos 2\theta\,[\sin 3\theta + \sin 5\theta]$	
	$= 2\cos 2\theta\,[2\sin 4\theta\cos\theta]$	B1 Factor formula
	$= 4\cos 2\theta\cos\theta\sin 4\theta$	
	$= 4\cos 2\theta\cos\theta\,[2\sin 2\theta\cos 2\theta]$	B1 Double angle
	$= 8\cos^2 2\theta\cos\theta\sin 2\theta$	B1 Double angle
	$= 8\cos^2 2\theta\cos\theta\,[2\sin\theta\cos\theta]$	
	$= 16\cos^2 2\theta\cos^2\theta\sin\theta$	
	$= 16\sin\theta\cos^2\theta\cos^2 2\theta$	B1

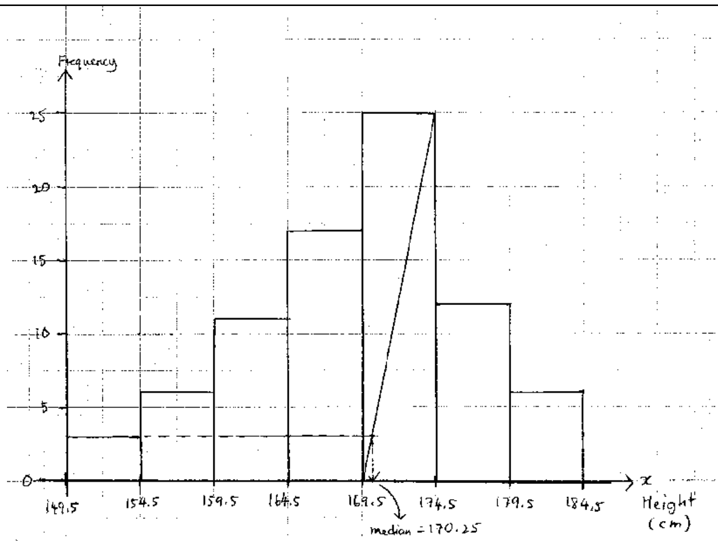
Q	Mark Scheme	Marks
2 (a)	∠PAB = ∠TPR = 65° [Angles in alternate segment are equal]	B1
	∠PRQ = ∠PAB = 65° [The exterior angles of a cyclic quadrilateral is equal to the opposite interior angle]	B1
		
	∠APB = 180° – (65° + 72°) [The sum of angles on straight line APK is 180°] = 43°	B1
	∠PBA = 180° – (65° + 43°) [The sum of angles in triangle APB is 180°] = 72°	
	∠PQR = ∠PBA = 72° [The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.]	B1

Q	Mark Scheme	Marks
2 (b) (i)	**∠PSR = ∠ PQS [Angles in alternate segments are equal]** **∠PQS = ∠OSQ or ∠OQS [Base angles of an isosceles triangle are the same. OQ = OS , radius of a circle .]** **∴ ∠PSR = ∠ OSQ** 	**B1** **B1** **B1**
2 (b) (ii)	**∠PRS = ∠QTS = 90° [All right angles are congruent .]** **Let ∠PSR = *x*** **∠SPR = 90° – *x* [The sum of angles in a triangle is 180° .]** **From (i) ∠OSQ = ∠PSR = *x*** **∠ QST = 90° – ∠OSQ = 90° – *x* [Tangent is perpendicular to the radius]** **∴ ∠SQT = *x* [The sum of angles in a triangle is 180° .]** **∴ ∠SPR = ∠QST = 90° – *x*** **∴ ∠PSR = ∠SQT = *x***  **All the corresponding angles are the same, therefore ΔPSR and ΔSQT are similar.**	**B1 equate 1st pair of angle** **B1 equate 2 pairs of angles** **B1**
2 (b) (iii)	**ΔPSR and ΔSQT are similar.** $\frac{PR}{ST} = \frac{PS}{QS} = \frac{RS}{QT}$ $\therefore \frac{PR}{ST} = \frac{RS}{QT}$ $\therefore PR \cdot QT = RS \cdot ST$	**M1** **A1**

Q	Mark Scheme	Marks
3 (a)	$\lambda\, i + 3\, j = k\,[\,3\, i + (8 + \lambda)\, j\,]$ $\lambda\, i + 3\, j = 3\, k\, i + k\,(8 + \lambda)\, j$ $\therefore\ 3\,k = \lambda$ $\qquad k\,(8 + \lambda) = 3$ $k = \frac{\lambda}{3}$ $\qquad \frac{\lambda}{3}(8 + \lambda) = 3$ $\lambda^2 + 8\,\lambda - 9 = 0$ $(\lambda + 9)\,(\lambda - 1) = 0$ $\lambda = -9$ or $\lambda = 1$	**B1** **with constant *k*** **M1** **2 equations** **A1**
3 (b)	$\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ}$ $= (3\,i - j) - (2\,i + 2\,j)$ $= i - 3\,j$ $\overrightarrow{RP} = \overrightarrow{OP} - \overrightarrow{OR}$ $= (3\,i - j) - (-i - 2\,j)$ $= 4\,i + j$	**B1 any one correct**
	$\overrightarrow{QP} \cdot \overrightarrow{RP} = (i - 3\,j)\,(4\,i + j)$ $= 1\,(4) + (-3)\,(1)$ $= 1$ $\cos\theta = \dfrac{\overrightarrow{QP} \cdot \overrightarrow{RP}}{\lvert\overrightarrow{QP}\rvert\ \lvert\overrightarrow{RP}\rvert}$ $\cos\theta = \dfrac{1}{\sqrt{1^2+3^2} + \sqrt{4^2+1^2}}$ $\cos\theta = \dfrac{1}{\sqrt{10}\ \sqrt{17}}$ $\theta = 85°\ 36'$	**B1** **Scalar product** **M1** **A1**

Q	Mark Scheme	Marks
4	$y = v x$, $\frac{dy}{dx} = x\left(\frac{dv}{dx}\right) + v$	B1
	$y\left(\frac{dy}{dx}\right) = 2y - x$ $v x\left[x\left(\frac{dv}{dx}\right) + v\right] = 2(v x) - x$ $v x^2\left(\frac{dv}{dx}\right) + v^2 x - 2 v x + x = 0$ $v x^2\left(\frac{dv}{dx}\right) + x(v^2 - 2v + 1) = 0$ $v x^2\left(\frac{dv}{dx}\right) + x(v-1)^2 = 0$ $x\left(\frac{dv}{dx}\right) + \frac{(v-1)^2}{v} = 0$	M1 substitute A1
	$x\left(\frac{dv}{dx}\right) = \frac{-(v-1)^2}{v}$ $\int_2^v \frac{v}{(v-1)^2}\, dv = -\int_1^x \frac{1}{x}\, dx$ $\int_2^v \frac{1}{(v-1)} + \frac{v}{(v-1)^2}\, dv = -\int_1^x \frac{1}{x}\, dx$ $\left[\ln(v-1) - \frac{1}{(v-1)}\right]_2^v = -[\ln x]_1^x$ $\left[\ln(v-1) - \frac{1}{(v-1)}\right] - [\ln 1 - 1] = -[\ln x - \ln 1]$ $\ln(v-1) - \frac{1}{(v-1)} + 1 = -\ln x$ $\ln\left(\frac{y}{x} - 1\right) - \frac{1}{\left(\frac{y}{x} - 1\right)} + 1 = -\ln x$ $\ln\left(\frac{y-x}{x}\right) - \frac{1}{\left(\frac{y-x}{x}\right)} + 1 = -\ln x$ $\ln\left(\frac{y-x}{x}\right) + \ln x = -1 + \left(\frac{x}{y-x}\right)$ $\ln\left(\frac{y-x}{x}\right) x = \frac{-(y-x)+x}{y-x}$ $\ln(y-x) = \frac{2x-y}{y-x}$	M1 separate M1 A1 correct partial fractions M1 correct integration M1 substitute limits M1 Substitute $v = \frac{y}{x}$ A1

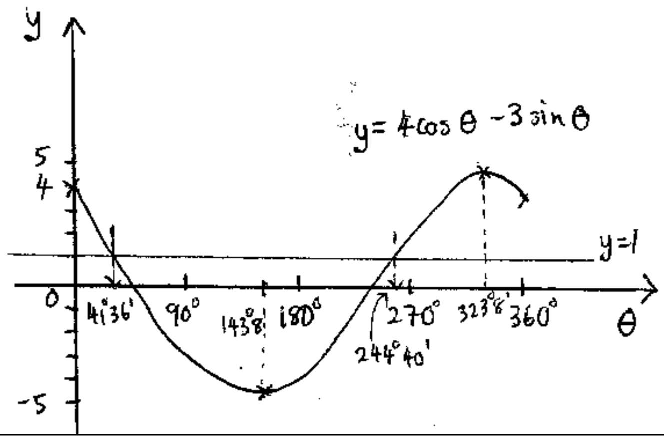
Q	Mark Scheme	Marks
5 (a)		**D1 Stemplot** **D1 Key**
5 (b)	**Median** $= \frac{35+36}{2}$ $\quad Q_1 = \frac{31+31}{2}$ $\quad Q_3 = \frac{41+41}{2}$ **Median = 35.5** $\quad Q_1 = 31$ $\quad Q_3 = 41$	**B1**
	Lower Boundary $= 31 - 1.5\,(41 - 31)$ $= 16$ **Upper Boundary** $= 41 - 1.5\,(41 - 31)$ $= 56$	**B1**
	 ∴ **Outlier is 60**	**D1 Box & whiskers** **B1**

Q	Mark Scheme	Marks
6 (a)	 From the histogram , median = 170.25 or 170.5	D1 correct scale D1 correct rectangles M1 draw line A1
(b)	mean = $\frac{13535}{80}$ mean = 169.1875 standard deviation = $\sqrt{\frac{2\,294\,295}{80} \quad \left(\frac{13535}{80}\right)^2}$ = 7.3673	M1 $\sum x$, A1 B1 $\sum x^2$ M1 A1
(c)	The mean and standard deviation are not the best statistical representation because this distribution is negatively skwed.	B1

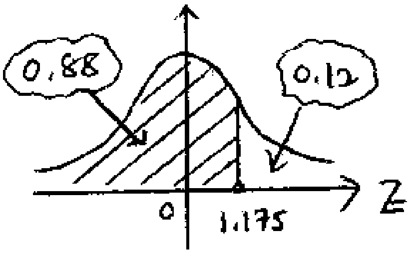
Q	Mark Scheme	Marks
7	$E(X) = \sum x\,P(X = x)$ $= 1\left(\frac{1}{n}\right) + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right) + \ldots + n\left(\frac{1}{n}\right)$ $= \left(\frac{1}{n}\right)[1 + 2 + 3 + \cdots + n]$ $= \left(\frac{1}{n}\right)\left[\frac{1}{2}n(n+1)\right]$ $= \frac{1}{2}(n+1)$	M1 A1
	$E(X^2) = \sum x^2\,P(X = x)$ $= 1^2\left(\frac{1}{n}\right) + 2^2\left(\frac{1}{n}\right) + 3^2\left(\frac{1}{n}\right) + \ldots + n^2\left(\frac{1}{n}\right)$ $= \left(\frac{1}{n}\right)[1^2 + 2^3 + 3^2 + \cdots + n^2]$ $= \left(\frac{1}{n}\right)\left[\frac{1}{6}n(n+1)(2n+1)\right]$ $= \frac{1}{6}(n+1)(2n+1)$	M1 A1
	$Var(X) = \frac{1}{6}(n+1)(2n+1) - \left[\frac{1}{2}(n+1)\right]^2$ $= \frac{1}{6}(n+1)(2n+1) - \frac{1}{4}(n+1)^2$ $= \frac{1}{12}(n+1)[2(2n+1) - 3(n+1)]$ $= \frac{1}{12}(n+1)[n - 1]$ $= \frac{1}{12}(n^2 - 1)$	M1 A1

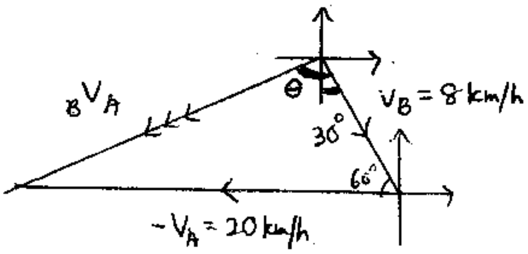
Q	Mark Scheme	Marks
8 (a)	$E(X) = \int_0^{\infty} x \left(\frac{1}{100} e^{-x/100}\right) dx$ $= \int_0^{\infty} \left(\frac{x}{100} e^{-x/100}\right) dx$ $= \left[\frac{x}{100}\left(-100 e^{-x/100}\right)\right]_0^{\infty} - \int_0^{\infty} \frac{1}{100}\left(-100 e^{-x/100}\right) dx$ $= \left[-x e^{-x/100}\right]_0^{\infty} + \int_0^{\infty}\left(e^{-x/100}\right) dx$ $= \left[-x e^{-x/100}\right]_0^{\infty} - \left[100 e^{-x/100}\right]_0^{\infty}$ $= \left[\frac{-x}{e^{x/100}}\right]_0^{\infty} - \left[\frac{100}{e^{x/100}}\right]_0^{\infty}$ $= 0 - \left[0 - \frac{100}{1}\right]$ $= 100$	M1 M1 By Parts M1 Correct integration M1 correct Limits A1
8 (b)	X represent the lifespan of one electrical component $Y = X_1 + X_2 + X_3 + X_4 + X_5$ $E(Y) = 5E(X)$ $= 5(100)$ $= 500$	B1

Q	Mark Scheme	Marks
9 (a)	$4\cos\theta - 3\sin\theta \equiv R\cos(\theta + \alpha)$ $4\cos\theta - 3\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $R\cos\alpha = 4$, $R\sin\alpha = 3$ | $\tan\alpha = \frac{3}{4}$, $\alpha = 36°\,52'$ | $R = \sqrt{4^2 + 3^2}$, $R = 5$ $\therefore\ 4\cos\theta - 3\sin\theta \equiv 5\cos(\theta + 36°\,52')$	M1 correct R & α A1
	$4\cos\theta - 3\sin\theta = 1$ $5\cos(\theta + 36°\,52') = 1$ $\cos(\theta + 36°\,52') = \frac{1}{5}$ $\theta + 36°\,52' = 78°\,28',\ 281°\,32'$ $\theta = 41°\,36',\ 244°\,40'$	M1 A1

9 (b)	$y = 4\cos\theta - 3\sin\theta$, $y = 5\cos(\theta + 36^\circ 52')$ y_{max} is 5 when $\theta + 36^\circ 52' = 360^\circ$ $\theta = 323^\circ 8'$ y_{min} is -5 when $\theta + 36^\circ 52' = 180^\circ$ $\theta = 143^\circ 8'$	 B1 B1
		D1 correct shape D1 max & min points
	The solution set is $\{\ \theta : 0^\circ \le \theta \le 41^\circ 36'$ or $244^\circ 40' \le \theta \le 360^\circ\ \}$	D1 draw line y = 1 B1

Q	Mark Scheme	Marks
10 (a)	X represents the number of ingots containing gold. $X \sim B(800, 0.005)$ $x = 0, 1, 2, 3, \ldots, 800$ <u>Poisson Approximation :</u> $\lambda = 4$ P (lucky month) = P ($X \ge 4$) $= 1 - [\ P(X=0) + P(X=1) + P(X=2) + P(X=3)\ ]$ $= 1 - e^{-4}\left[1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!}\right]$ $= 1 - \frac{71}{3}e^{-4}$ $= 0.567$ (correct to 3 significant figures).	 B1 B1 M1 A1
10 (b)	Y represents the number of lucky months. $Y \sim B(24, 0.567)$ $y = 0, 1, 2, 3, \ldots, 24$ <u>Normal Approximation :</u> mean = 13.608 , variance = 5.892264 $P(Y > 12) = P\left(Z > \frac{12.5 - 13.608}{\sqrt{5.892264}}\right)$ $= P(Z > -0.456)$ $= 1 - 0.3242$ $= 0.6758$	 B1 B1 M1 continuity correction M1 standardize A1

Q	Mark Scheme	Marks
11 (a)	$X \sim N(0.95, \sigma^2)$ $P(X < 0.98) \geq 0.88$ $P\left(Z < \frac{0.98 - 0.95}{\sigma}\right) \geq 0.88$ $\frac{0.03}{\sigma} \geq 1.175$ $\frac{0.03}{1.175} \geq \sigma$ $\sigma \leq 0.0255$ 	B1 M1 standardize M1 A1
11 (b) (i)	Four runners A , B , C , D ran 100 m $A, B, C, D \sim N(19, 0.2^2)$ One runner , $E \sim N(58, 1.0^2)$ $E(E - 4A) = 58 - 4(14) = 2$ $Var(E - 4A) = 1^2 + 16(0.2^2) = 1.64$ $P(E < 4A) = P(E - 4A < 0)$ $= P\left(Z < \frac{0 - 2}{\sqrt{1.64}}\right)$ $= P(Z < -1.562)$ $= 0.0592$	B1 correct mean & variance B1 …P (E < 4A) M1 standardize A1
11 (b) (ii)	$E(A + B + C + D) = 4(14) = 56$ $Var(A + B + C + D) = 4(0.2^2) = 0.16$ $E[E - (A + B + C + D)] = 58 - 56 = 2$ $Var[E - (A + B + C + D)] = 1^2 + 0.16 = 1.16$ $P(E - (A + B + C + D) < 3) = P\left(Z < \frac{3 - 2}{\sqrt{1.16}}\right)$ $= P(Z < -0.928)$ $= 1 - 0.1768$ $= 0.8232$	B1 correct mean & variance B1 … P (…< 3) M1 standardize A1

<table>
<tr><th>Q</th><th>Mark Scheme</th><th>Marks</th></tr>
<tr><td>12
(a)</td><td>


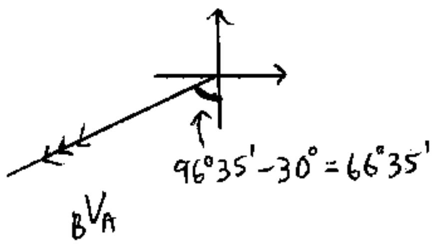
$_BV_A{}^2 = 8^2 + 20^2 - 2(8)(20)\cos 60°$

$_BV_A{}^2 = 304$, $_BV_A = 17.4356$ or $_BV_A = \sqrt{304}$

The velocity of B relative to A is 17.44 km/h .
</td><td>D1
diagram

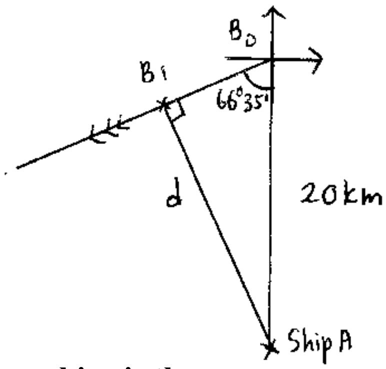
B1</td></tr>
<tr><td></td><td>
$\dfrac{\sqrt{304}}{\sin 60°} = \dfrac{20}{\sin\theta}$

$\sin\theta = \dfrac{20\sin 60°}{\sqrt{304}}$



Acute angle = 83° 25'
θ = 180 ° – 83° 25'
= 96° 35'
The velocity of B relative to A is in the direction of
S 66° 35' W or [246° 35']
</td><td>B1</td></tr>
<tr><td>12
(b)

12
(c)</td><td>
The closest distance between A and B = 20 sin 66° 35'
= 18.35 km



Time taken = $\dfrac{20\cos 66°35'}{\sqrt{304}}$

= 0.4559 hour

= 27.35 minutes

Time when the distance between the two ships is the closest is 12.27 pm.
</td><td>M1
A1

M1

A1</td></tr>
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